

# Evolution and Dynamics of Cusped Light-Like Wilson Loops in Loop Space

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## Abstract

We discuss the possible relation between the singular structure of TMDs on the light-cone and the geometrical behaviour of rectangular Wilson loops.

## 1 Introduction

Transverse momentum dependent parton density functions (or TMDs for short) are known to have a more complex singularity structure than collinear parton density functions. Common singularities like ultraviolet poles can be removed by general methods like standard renormalisation using the  $R$ -operation. In the case of a light-like TMD however, where at least one of its segments is on the light-cone, it is not entirely clear whether standard renormalisation remains a sufficient technique, due to the emergence of extra overlapping divergencies. A standard TMD can be defined as [1]:

$$f(x, \mathbf{k}_\perp) = \frac{1}{2} \int \frac{dz^- d^2 \mathbf{z}_\perp}{2\pi(2\pi)^2} e^{ik \cdot z} \langle P, S | \bar{\psi}(z) U^\dagger(z; \infty) \gamma^+ U(\infty; 0) \psi(0) | P, S \rangle \Big|_{z^+=0} \quad (1)$$

where the Wilson lines are split into their longitudinal and transversal parts:

$$U(\infty, 0) = U(\infty^-, \infty_\perp; \infty^-, \mathbf{0}_\perp) U(\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp) \quad (2)$$

$$= \mathcal{P} e^{-ig \int_0^\infty dz_\perp A_\perp(\infty^-, \mathbf{z}_\perp)} \mathcal{P} e^{-ig \int_0^\infty dz^- A^+(z^-, \mathbf{0}_\perp)} \quad (3)$$

When on light-cone, this TMD will possess extra divergencies proportional to  $\frac{1}{\epsilon^2}$  (when using dimensional regularisation). These will give the only contribution to the evolution equations, governed by the cusp anomalous dimension [2, 3]

$$\Gamma_{\text{cusp}} = \frac{\alpha_s C_F}{\pi} (\chi \coth \chi - 1) \xrightarrow{\text{on-LC}} \frac{\alpha_s C_F}{\pi} \quad (4)$$

where  $\chi$  is the cusp angle (in literature sometimes referred to as a ‘hidden cusp’) which goes to infinity in the light-cone limit. In the next sections, we will show that a specific type of Wilson loop, namely rectangular loops with light-like segments on the null-plane, has its singularity structure analogous to on-LC TMDs, which feeds the idea that there might exist a duality between those two objects.

## 2 Wilson Loops as Elementary Objects in Loop Space

A general Wilson loop is defined as

$$\mathcal{W}[C] = \frac{1}{N_c} \text{tr} \langle 0 | \mathcal{P} e^{ig \oint_C dz^\mu A_\mu^a(z) t_a} | 0 \rangle \quad (5)$$

where  $C$  is any closed path and  $A_\mu$  is taken in the fundamental representation. This loop is a pure phase, traced over Dirac indices and evaluated in the ground state, transforming coordinate dependence into path dependence. As is known (see [4, 5]), Wilson loops can be used as elementary objects to completely recast QCD in loop space. To achieve this, the definition of a Wilson loop needs to be extended to make it dependent on multiple contours:

$$\mathcal{W}_n(C_1, \dots, C_n) = \langle 0 | \Phi(C_1) \dots \Phi(C_n) | 0 \rangle \quad \Phi(C) = \frac{1}{N_c} \text{tr} \mathcal{P} e^{ig \oint_C dz^\mu A_\mu(z)} \quad (6)$$

All gauge kinematics are encoded in a  $\mathcal{W}_1$  loop, and all gauge dynamics are governed by a set of geometrical evolution equations, the Makeenko-Migdal equations [6]:

$$\partial^\nu \frac{\delta}{\delta \sigma_{\mu\nu}(x)} \mathcal{W}_1(C) = g^2 N_c \oint_C dz^\mu \delta^{(4)}(x - z) \mathcal{W}_2(C_{xz} C_{zx}) \quad (7)$$

where two (geometrical) operations are introduced, namely the path derivative  $\partial_\mu$  and the area derivative  $\frac{\delta}{\delta \sigma_{\mu\nu}(x)}$  [6]. Although the Makeenko-Migdal equations provide an elegant method to describe the evolution of a generalised Wilson loop solely in function of its path, they have their limitations. For starters, they are not closed since the evolution of  $\mathcal{W}_1$  depends on  $\mathcal{W}_2$ . Formally, this limitation is superfluous in the large  $N_c$  limit since then we can make use of the ‘t Hooft factorisation property  $\mathcal{W}_2(C_1, C_2) \approx \mathcal{W}_1(C_1) \mathcal{W}_1(C_2)$  [6], making the MM equations closed. The remaining limitations of the MM equations are more severe. For one, the evolution equations are derived by applying the Schwinger-Dyson methodology on the Mandelstam formula

$$\frac{\delta}{\delta \sigma_{\mu\nu}(x)} \Phi(C) = ig \text{tr} \{ F^{\mu\nu} \Phi(C_x) \} \quad (8)$$

and using the Stokes’ theorem. These might, as well as the area derivative, not be well-defined for all types of paths. In particular, all contours containing one

or more cusps might induce some problematic behaviour, as it is (at least) not straightforward to define *continuous* area differentiation inside a cusp, nor it is to continuously deform a contour in a general topology [7]. This is somewhat bothersome, as most interesting dynamics lies in contours with cusps.

### 3 Evolution of Rectangular Wilson Loops

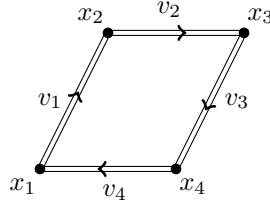


Figure 1: Parametrisation of a rectangular Wilson loop in coordinate space.

Now we turn to a specific type of path, namely a rectangular path with light-like segments ( $v_i^2 = 0$ ) on the null-plane ( $\mathbf{x}_\perp = 0$ ), as depicted in Figure (3). To investigate its singularity structure, we evaluate the loop (5) at one loop in coordinate space [8]:

$$\mathcal{W}_{\text{L.O.}} = 1 - \frac{\alpha_s C_F}{\pi} (2\pi\mu^2)^\epsilon \Gamma(1-\epsilon) \left[ \frac{1}{\epsilon^2} \left(-\frac{s}{2}\right)^\epsilon + \frac{1}{\epsilon^2} \left(-\frac{t}{2}\right)^\epsilon - \frac{1}{2} \ln^2 \frac{s}{t} \right] \quad (9)$$

where  $s$  and  $t$  are the Mandelstam energy/rapidity variables (note the positive sign in  $t$ ):

$$s = (v_1 + v_2)^2 \quad t = (v_2 + v_3)^2 \quad v_i = x_i - x_{i+1} \quad (10)$$

Note the  $\frac{1}{\epsilon^2}$  poles, which are the overlapping divergencies that stem from the light-like behaviour of the contour segments. The fact that they appear already at leading order renders this kind of Wilson loop non-renormalisable (at least not using the standard  $R$ -operation). The most straightforward way to manage them is by deriving an evolution equation for the loop. This is done by double differentiation (after rescaling  $\bar{s} = \pi e^{\gamma_E} \mu^2 s$ ):

$$\frac{d}{d \ln \mu} \frac{d}{d \ln s} \mathcal{W}_{\text{L.O.}} = -2 \frac{\alpha_s C_F}{\pi} = -2\Gamma_{\text{cusp}} \quad (11)$$

where we recognise the cusp anomalous dimension in the light-cone limit from (4). Thus, as anticipated in the TMD case, the only contribution to the evolution equations stems from the overlapping divergencies. Their concurrent appearance in the on-LC TMD case and in the case of an on-LC rectangular Wilson loop again hints to the existence of a duality between both.

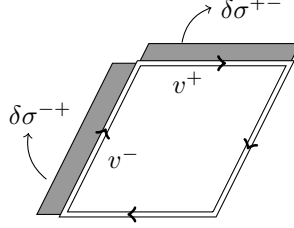


Figure 2: Angle-conserving deformations of a light-like rectangular loop on the null-plane.

## 4 Geometrical Behaviour and Relation to TMDs

In an attempt to combine the geometrical approach of the Makeenko-Migdal method with the evolution equations at leading order just derived, we investigate area differentiation on rectangular light-like loops on the null-plane, rendering the area differentials well-defined (see Figure (2)) [7]. This gives  $\delta\sigma^{+-} = \oint dx^- x^+ = v^+ \delta v^-$  and  $\delta\sigma^{-+} = \oint dx^+ x^- = v^- \delta v^+$ . Next we introduce the area variable  $\Sigma$ :

$$\Sigma \equiv v^- \cdot v^+ = \frac{1}{2}s \quad \frac{\delta}{\delta \ln \Sigma} = \sigma_{\mu\nu} \frac{\delta}{\delta \sigma_{\mu\nu}} \quad (12)$$

Replacing  $s$  by  $\Sigma$  in equation (11) gives  $-4\Gamma_{\text{cusp}}$ . Motivated by this, we conjecture a general evolution equation for light-like rectangular Wilson loops:

$$\frac{d}{d \ln \mu} \left[ \sigma_{\mu\nu} \frac{\delta}{\delta \sigma_{\mu\nu}} \ln \mathcal{W} \right] = - \sum_i \Gamma_{\text{cusp}} \quad (13)$$

Besides for light-like rectangular Wilson loops, equation (13) is expected to be valid for light-like TMDs, as they possess the same singularity structure. The area variable then gets replaced by the rapidity variable. This gives

$$\frac{d}{d \ln \mu} \frac{d}{d \ln \theta} f(x, \mathbf{k}_\perp) = 2\Gamma_{\text{cusp}} \quad (14)$$

The minus disappeared because  $\theta \sim \Sigma^{-1}$  ( $\theta = \frac{\eta}{p \cdot v^-}$  and  $p \sim v^+$ , so  $\theta \sim (v^+ v^-)^{-1}$ ), and there is a factor 2 since we have two (hidden) cusps. This result is very similar to the Collins-Soper evolution equations for off-LC TMDs.

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